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A BALLISTIC MISSILE DEFENSE MODEL FOR URBAN TARGETS

By

Kenneth Charles Shumate

United States Naval Postgraduate School

THESIS

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DEFENSE MODEL FOR URBAN TARGETS

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A Ballistic Missile

Defense Model for Urban Targets

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis presents a ballistic missile defense allocation model for the terminal defense of urban targets of varied value. The model allocates interceptors in proportion to the value of targets. Defensive missiles have a probability of interception, offensive re-entry vehicles are perfect, and the offense knows both the defensive allocation and firing doctrine. The area defended by a single interceptor farm is considered to be a point target and can be defended by no other interceptors. For any value of the offensive payoff in expected value per re-entry vehicle, the model determines the least cost minmax allocation and firing doctrine.

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I. INTRODUCTION

This thesis considers the terminal defense of a set of urban point targets of varied value in which the role of the defense is to minimize damage during a large attack by an offensive force of nuclear ballistic missiles. The attack is assumed to be sequential. The offense receives no information concerning the destruction of any target, and the defense does not know the size of the attack at any target. Specifically considered is the minmax allocation of interceptor missiles based on the assumption that the offense knows both the allocation of interceptors and the interceptor commitment policy at each target. This analysis ignores interactions with area defenses and assumes a constant single shot kill probability $p < 1$ associated with one interceptor against one re-entry vehicle. Branch [1] considers a similar problem but makes different simplifying assumptions. Shaver [2] considers a radar defense problem that is closely related to the defense of urban point targets. Shaver's objective function for the defense is to maximize the expected number of re-entry vehicles engaged. Battle [3] considers city defense and minimizes the maximum average damage per attacker. He drops the point target assumption but does not consider firing doctrine.

An important concept for ballistic missile defense studies is that of the price of a target. Price is defined to be the number of re-entry vehicles r sent against a target, divided by the probability K that the target is killed:

price = r/K. One can speak of the price that the offense "pays" to "buy" the target. The offensive payoff λ at a single target is the value V of the target times the reciprocal of price: $\lambda = KV/r$. $\lambda = \frac{\text{Value of Tgt}}{\text{Price of Tgt}}$

A proportional defense is one in which the defense — i.e. more defense for higher value tgt. — forces the offense to pay a price that is proportional to the value of a target. For certain sets of targets, particularly those in which many defended targets are not attacked by the optimal offense, proportional defense will minimize expected target damage for a fixed force of interceptors. It is assumed that proportional defense is optimal in this sense for the set of targets under consideration. The method of requiring price to be proportional to value will be to allocate interceptors in such a manner that the

λ = Value of Tgt / Price of Tgt = constant for each tgt

offensive payoff is the same at each target. Rather than allocate a fixed force of interceptors, the model minimizes the number of interceptors required to force the offensive payoff to be less than or equal to a fixed payoff λ^*.

II. THE ALLOCATION PROBLEM

A. OFFENSE

The offense must allocate a fixed force $\hat{R}$ of re-entry vehicles (R.V.'s) to a set of N targets of varied value in such a way as to maximize total expected fatalities. Suppose the damage functions $f_i(r_i)$, $i = 1,\ldots,N$ give the expected fatalities at the i^{th} target for an attack of r_i R.V.'s. The offensive problem is:

$$\max_{\overline{R}} \sum_{i=1}^{N} f_i(r_i)$$

subject to

$$\sum_{i=1}^{N} r_i = \hat{R}$$

$$r_i \geq 0 \qquad i=1,\ldots,N$$

where $\overline{R} = (r_1, r_2, \ldots, r_N)$, the offensive allocation. It is assumed that the f_i are continuous and differentiable. From the Kuhn-Tucker conditions or directly from Gibb's Lemma, there exists a λ^0 such that:

$$\frac{df_i(r_i^0)}{dr} = \lambda^0 \;, \quad r_i^0 > 0$$

$$\leq \lambda^0 \;, \quad r_i^0 = 0$$

where r_i^0 is the optimal attack at the i^{th} target. That is, the marginal value is the same at each target that is attacked. Note that λ^0 is the slope of $f_i(r_i)$ at $r_i = r_i^0$.

See Figure 1. The optimal offensive solution requires λ^0 to be the maximum marginal return obtainable subject to $\Sigma r_i = \hat{R}$.

Suppose a target is defended by I_i interceptors with single shot kill probability of <u>one</u>. i.e. $r_i - I_i$ are left unintercepted to attack the tgt! The damage function for a defended target is $f_i(r_i - I_i)$ for $r_i \geq I_i$. See Figure 2. Define

$$\lambda_i = \frac{f_i(r_i)}{r_i} \quad \text{and } \lambda_i^m = \max_{r_i} \frac{f_i(r_i)}{r_i}$$

Note that λ_i^m is the maximum offensive payoff at target i, in terms of expected fatalities per R.V. This value λ_i^m is the slope of the ray from the origin tangent to f_i. See Figure 2.

Consider a ray from the origin with slope λ^0. See Figure 3. Given the value of λ^0, the offense finds the optimal attack size against each target by finding r_i to $\max_{r_i} (f_i(r_i - I_i) - \lambda^0 r_i)$. Thus after having found λ^0, the offense can find an optimal solution to the overall problem by suboptimizing at each target.

B. DEFENSE

Since the defense assumes that the offense knows the target damage function $f_i(r_i)$ and the interceptor allocation $\bar{I} = (I_1, \ldots, I_N)$, the defensive problem for a fixed interceptor stockpile $\hat{I}$ is:

$$\min_{\bar{I}} \{ \max_{\bar{R}} \sum_i f_i(r_i) \}$$

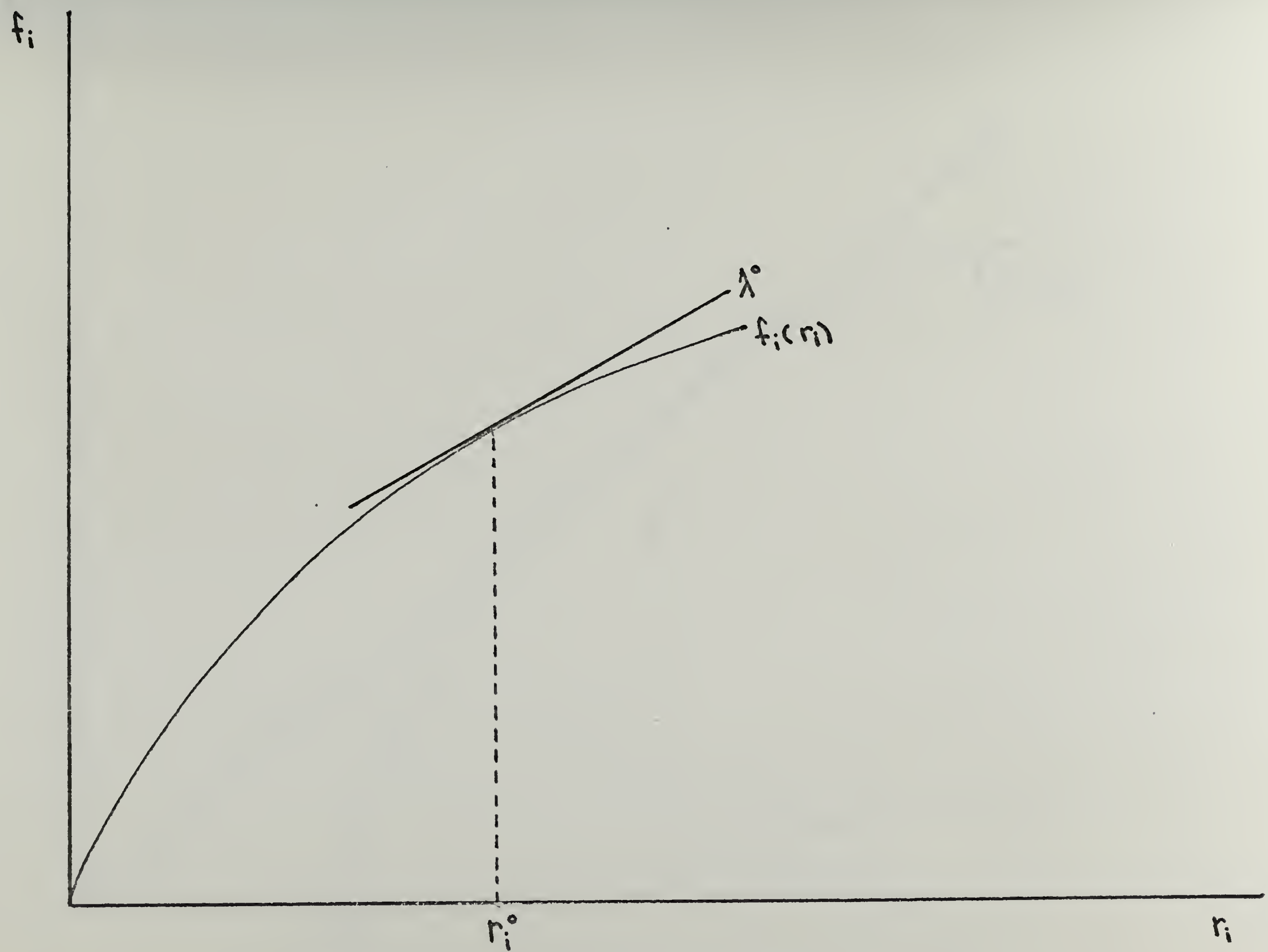

Figure 1. The Damage Function $f_i(r_i)$ and λ^0.

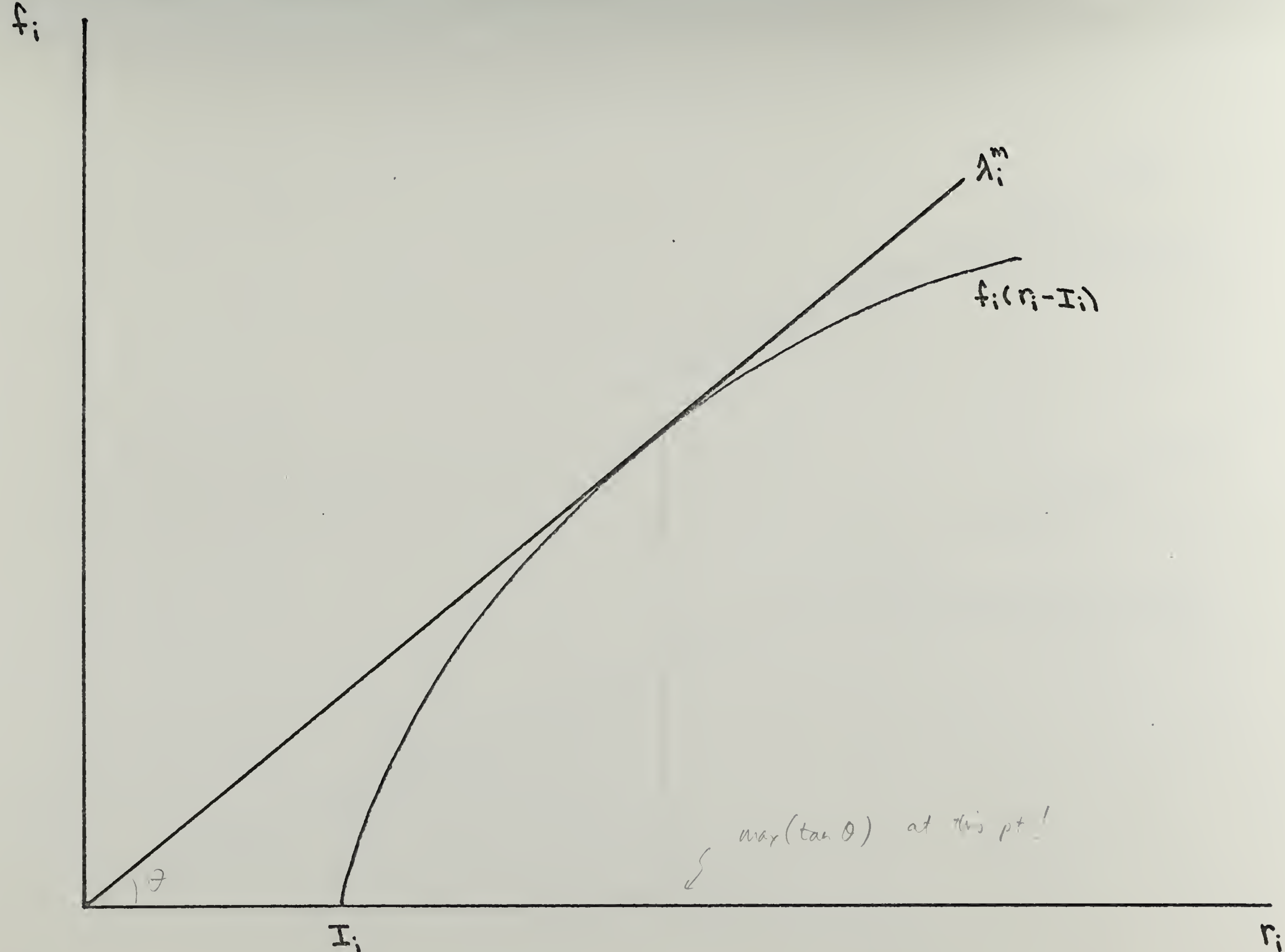

Figure 2. The Shifted Damage Function and λ_i^m.

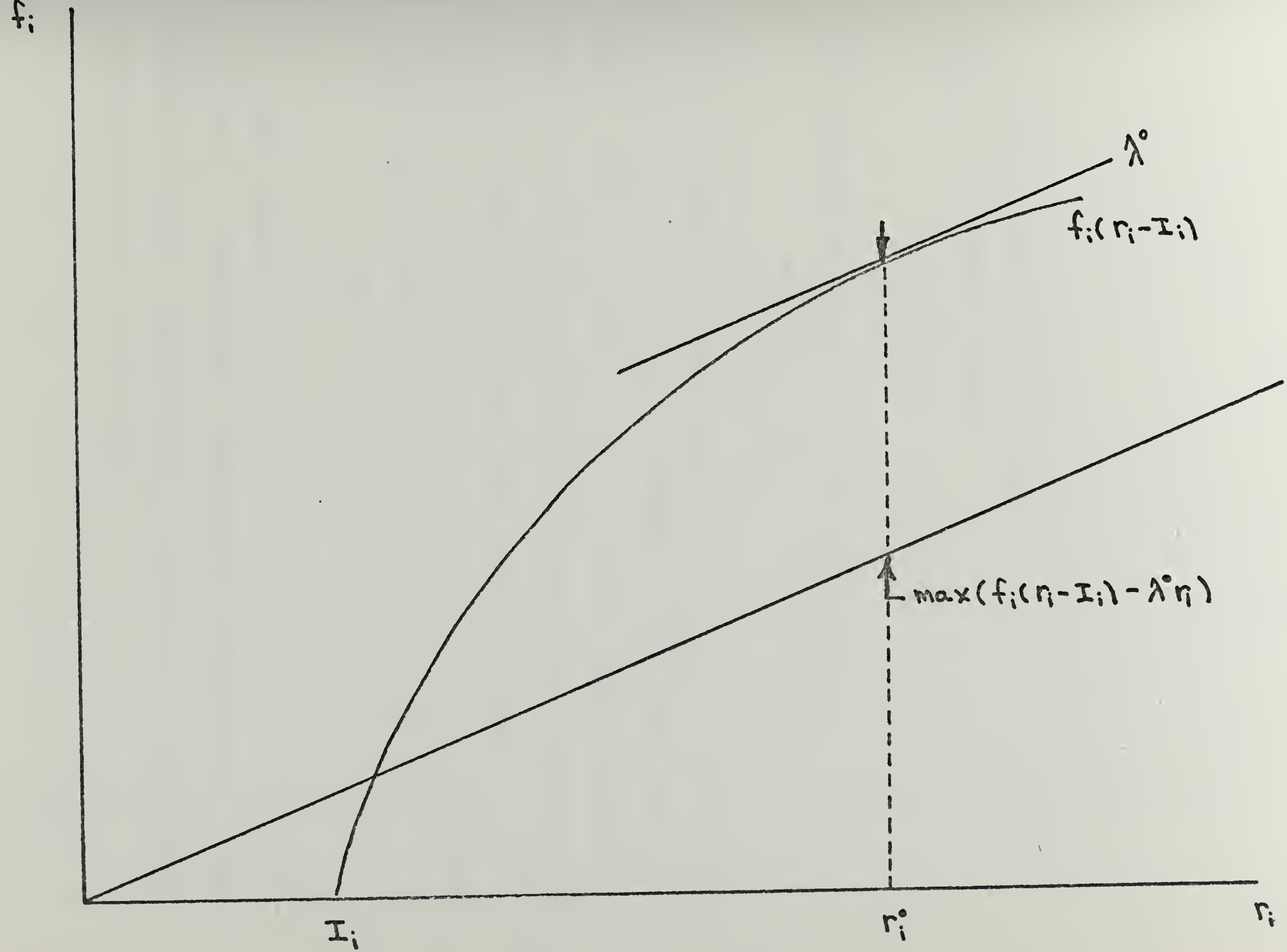

Figure 3. Max $(f_i(r_i) - \lambda^0 r_i)$.

$$\text{subject to} \quad \sum_i r_i = \hat{R}$$

$$\sum_i I_i = \hat{I}$$

$$I_i,\ r_i \geq 0 \quad i=1,\ldots,N\ .$$

Recall that for fixed $\overline{I}$ the offense was able to Max $\Sigma f_i(r_i)$ by suboptimization,

$$\underset{r_i}{\text{Max}}\ (f_i(r_i - I_i) - \lambda^0\ r_i).$$

The corresponding defensive suboptimization is

$$\underset{I_i}{\text{Min}}\ \{\ \underset{r_i}{\text{Max}}\ (f_i(r_i - I_i) - \lambda^0\ r_i)\}\ .$$

It is not generally true, however, that suboptimization will yield an optimal allocation for problems in which minmax > maxmin (see for example, [Ref. 4]), but if many defended targets are not attacked the suboptimization yields the optimal allocation. A proportional defense, in which λ_i^m is equal to a fixed λ^* for all i, accomplishes the suboptimization. Such a defense forces $\lambda^0 = \lambda_i^m = \lambda^*$, and then

$$\underset{r_i}{\text{Max}}\ (f_i(r_i - I_i) - \lambda^*\ r_i) = 0.$$

The remainder of this thesis develops a model for determining a proportional allocation of interceptors for the defense of point targets when the interceptor single shot kill probability is less than 1.

III. THE DEFENSIVE MODEL

Recall that p is the interceptor single shot kill probability and define $q = 1 - p$, the probability that a re-entry vehicle (R.V.) survives an encounter with a single interceptor. Define i_m = the number of interceptors sent against the m^{th} R.V. Thus the probability that the m^{th} R.V. is killed is $(1-q^{i_m})$ and the probability that the target is killed for an attack of size r is:

$$K = 1 - \prod_{m=1}^{r} (1-q^{i_m}).$$

Recall the offensive payoff λ at a single target is $\lambda = KV/r$. It is assumed that at each city the offense desires to $\max_r KV/r$.

The method used to arrive at proportional defense is to minimize the number of interceptors I allocated to a target, subject to $\lambda \leq \lambda^*$. Suppose that the defense fires against some number M of R.V.'s, so that the problem at each target is:

$$\text{Min} \quad I = \sum_{m=1}^{M} i_m$$

$$\text{s.t.} \quad KV/r \leq \lambda^* \qquad r = 1,2,\ldots$$

An offensive strategy is a choice of r, and a defensive strategy is a choice of a sequence of i_m's called the firing doctrine and denoted $FD = (i_1, i_2, \ldots, i_M)$. It is assumed for convenience that the defense never fires more than three interceptors at an R.V. Thus the defense must choose a number

B such that $i_m = 3$, $m = 1,2,\ldots,B$: a number A such that $i_m = 2$, $m = B+1,\ldots,A$: and an M such that $i_m = 1$, $m = A+1,\ldots,M$. Of course A and B may be zero. Notice that for a fixed firing doctrine K depends only on r. Since the offense knows the defensive firing doctrine, the defense must determine the firing doctrine that minimizes I and such that $\min_{FD} \{\max_{r} KV/r\} \leq \lambda^*$.

IV. ANALYSIS

A. INTRODUCTORY CONCEPTS

A special case will serve to introduce some basic concepts. Consider a target of value V. Suppose that I, the number of interceptors at the target, is arbitrarily large but that the firing doctrine calls for firing only one interceptor at each R.V. The firing doctrine is not optimal, but illustrative due to its simplicity. Define the payoff curve, L to be L = KV. Notice that since FD and p are fixed, K and hence L depend only on r. See Figure 4. The offensive payoff for an attack of size r is the slope of the ray, called the λ-line, from the origin to the curve L(r). The slope of the λ-line is KV/r and thus corresponding to each r there is a λ-line, designated $\lambda(r)$. Note that $\lambda(r)$ is not the expected value obtained by the r^{th} R.V. but is the average expected value per R.V. for an attack of size r: $\lambda(r) = KV/r$. Now note that $K = 1 - \prod_{m=1}^{r} (1-q^{i_m})$ is strictly concave in r for i_m constant. Thus $L(r) = K(r)V$ is strictly concave and $\lambda(r) > \lambda(r+1)$. That is, the offense receives decreasing marginal and average returns for increasing r when i_m is constant. Thus to maximize KV/r the optimal attack r^0 for the target and firing doctrine described is to fire one R.V. Thus $r^0 = 1$ and $\lambda(r^0) = KV/r$ where $K = 1 - p$. Then $\lambda = \lambda(r^0) = (1-p)V$ is the payoff at this target.

Define the λ*-line to be a ray from the origin with slope λ^*. Now recall that the defense's constraint is to

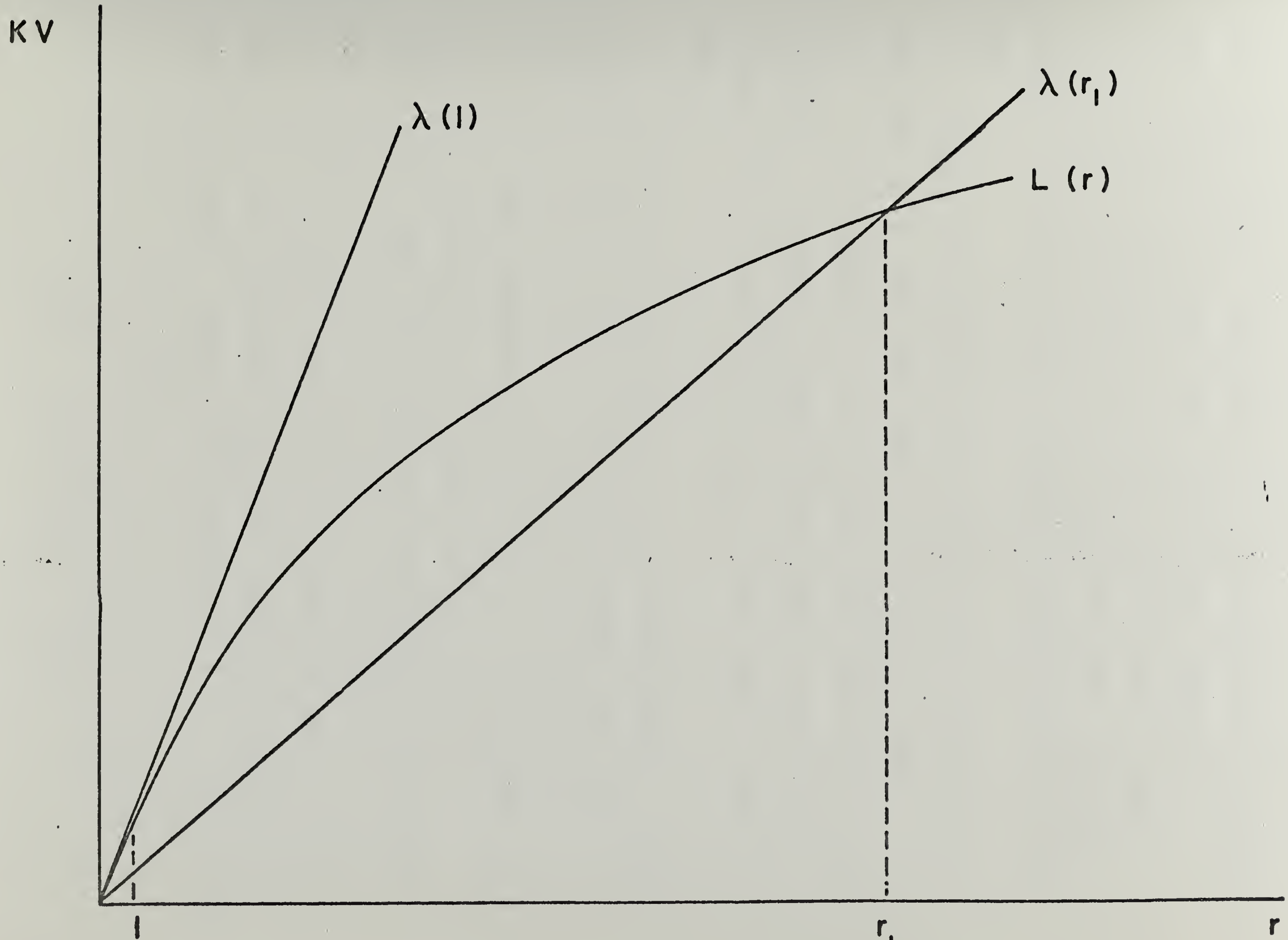

Figure 4. L(r) and λ-line.

force $\lambda \le \lambda^*$, i.e., $KV/r \le \lambda^*$. Graphically, the defense must force the slope of the λ-line to be less than λ^* for any size attack. Alternatively, the defense will be satisfied if $L(r)$ is always below the λ^*-line. That is, $L(r) \le \lambda^* r$. In Figure 2, L_1 is a feasible payoff curve since $L_1(r) \le \lambda^* r$ and L_2 is not feasible since there are values of r such that the offensive payoff is greater than λ^*.

Leaving the special case, consider the criterion for defending a target. Note that if a target has value $V \le \lambda^*$, $r \ge 1$ yields $\lambda \le \lambda^*$ and thus the target will not be defended. A target of value greater than λ^* must be defended since if undefended, $r = 1$ yields a payoff greater than λ^*.

Consider the situation at any defended target. Define $R = V/\lambda^*$ where V is the value of the target. See Figure 5. For an attack of size $r \ge R$, the value of the λ^*-line is $\lambda^* r \ge V$. Since maximum value of $L(r)$ is V, it is clear that for $r \ge R$ the constraint is always satisfied, i.e., $L(r) \le \lambda^* r$. Thus the defense will never fire at more than $R - 1$ R.V.'s. If the defense were to fire at fewer than $R - 1$ R.V.'s, the $R - 1^{st}$ (or an earlier one, since the offense does not have shoot-look-shoot capability) would destroy the target. Then $\lambda = KV/r$ where $K = 1$, $r = R - 1$, and $R > 1$ since $V > \lambda^*$. Thus:

$$\lambda = \frac{V}{(R-1)} = \frac{V}{[(V/\lambda^*)-1]} = \lambda^* \frac{V}{V-\lambda^*} > \lambda^*$$

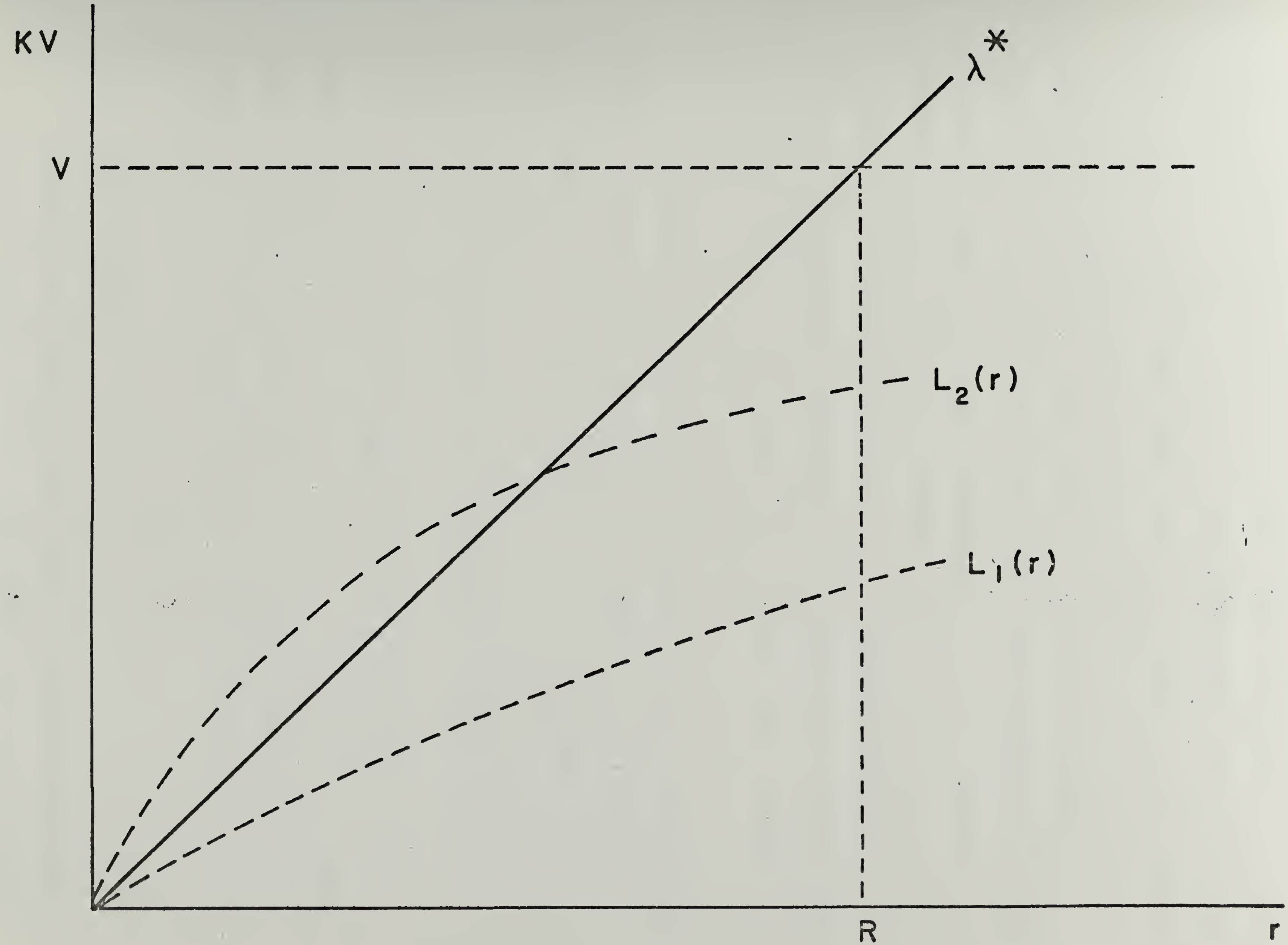

Figure 5. The λ^*-line and R.

For feasibility the defense must fire against at least R - 1 R.V.'s and for optimality it must fire at exactly R - 1 R.V.'s. An attack of size $r = R$ is called <u>exhausting</u>. For an exhausting attack, the offense will receive a payoff of λ^*. Since the defense will choose a firing doctrine such that the offense never receives more than λ^*, an optimal offensive strategy must be to exhaust the interceptor supply; $r^0 = R$. However, since fractional interceptors or R.V.'s are not allowed, R must be integer valued, but V/λ^* is not generally integer. Integer considerations are ignored during the analysis for ease of exposition, but numerical calculations use $R = [(V/\lambda^*) + .999]$ where $[x]$ is the largest integer in x.

In summary, note that the offense receives decreasing marginal returns for constant i_m, targets are defended if and only if $V > \lambda^*$, $M = (V/\lambda^*) - 1$, and an optimal offensive strategy is $r^0 = V/\lambda^*$. At any defended target, the only remaining problem is to determine a firing doctrine such that $\lambda \leq \lambda^*$.

Three classes of targets will be considered: small targets, for which $A = B = 0$, that is it suffices to send one interceptor against each R.V.; medium targets, which require $A > 0$; and large targets, which require $B > 0$ and $A > 0$.

B. SMALL TARGETS

Consider a target of small value in relation to p and λ^* so that; $(1-p)V \leq \lambda^* < V$. Then $FD^0 = (i_1, i_2, \ldots, i_{R-1})$;

$i_m = 1$, $m = 1,2,\ldots,R-1$ is optimal with $I = \sum_{i=1}^{R-1} i_m = R - 1$. This is shown by noting that FD^0 is feasible since $r = 1$ yields $\lambda = (1-p)V$ which is less than or equal to λ^* by assumption, and the offense receives decreasing returns for $r \leq R - 1$. Since all $i_m = 1$, FD^0 is certainly least cost and the defense must fire at $R - 1$ R.V.'s. Thus FD^0 is optimal. Figure 6 shows the payoff curve for this example. The offense may be considered to be facing this payoff curve when making an allocation of R.V.'s to targets, and it is clear graphically that the optimal attack is $r^0 = R$. If $(1-p)V = \lambda^*$ an alternate offensive optimum is $r^0 = 1$. In either case, r^0 yields λ^*.

C. MEDIUM TARGETS

In the example above the firing doctrine or commitment rule was to fire one interceptor at each of the first $R - 1$ R.V.'s. Consider another target (or a different λ^* or p) such that $(1-p)^2V \leq \lambda^* < (1-p)V$. If the defense maintains the same firing doctrine, the resulting payoff curve is shown in Figure 7. Now the offense receives $\lambda > \lambda^*$ for $r < r_1$, and the optimal attack is $r^0 = 1$, yielding $\lambda = (1-p)V > \lambda^*$. The value of this target is too high in relation to λ^* for the defense to fire only one interceptor at the first R.V. Recall that the defense must still fire at exactly $R - 1$ R.V.'s. Suppose the firing doctrine $FD = (i_1,\ldots,i_{R-1})$ $i_m = 2$, $m = 1,2,\ldots,A$; $i_m = 1$, $m = A+1,\ldots,R-1$. Consider the case $A = 1$. If $(1-p)V$ is only slightly greater than λ^* this doctrine would be feasible,

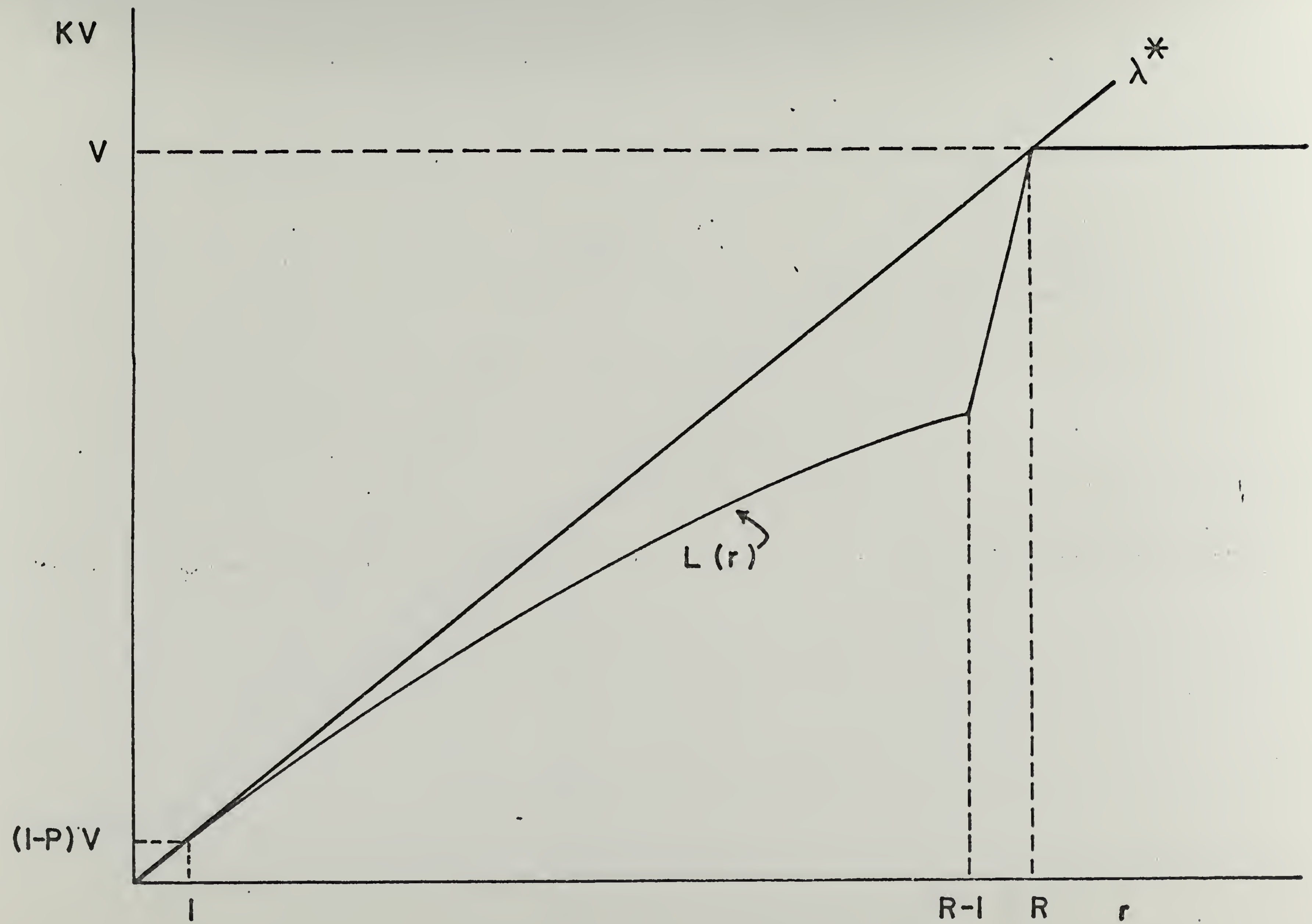

Figure 6. Payoff Curve for Small Target.

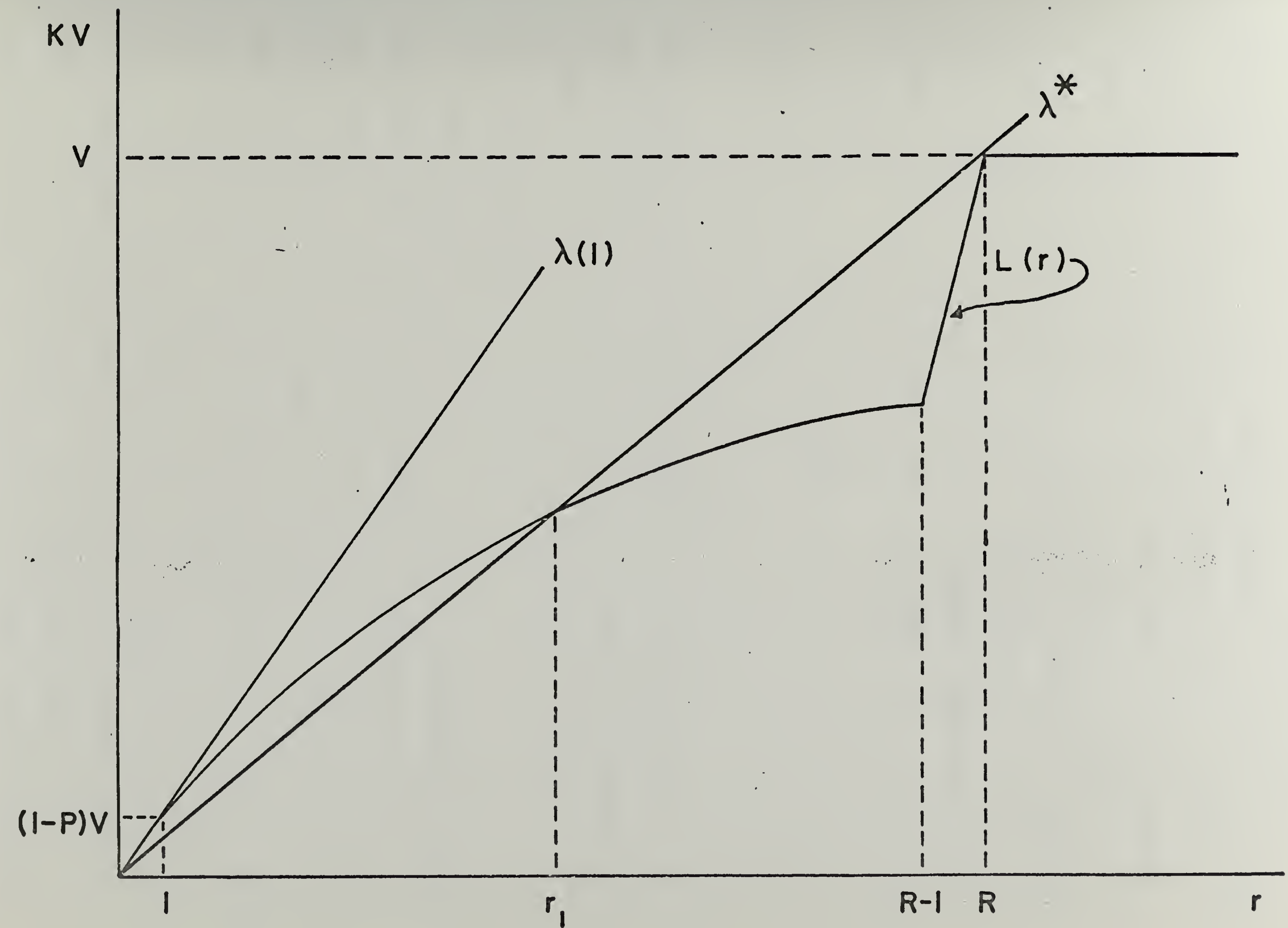

Figure 7. Non-Feasible Payoff Curve for Medium Target.

but in general the payoff curve is as shown in Figure 8. Now for $r_1 < r < r_3$ the offense receives $\lambda > \lambda^*$ and $r^0 = r_2$.

Consider the firing doctrine with $A = R - 1$. This firing doctrine must be feasible since $K(1) = 1-(1-(1-p)^2) = (1-p)^2$ and $\lambda(1) = (1-p)^2V < \lambda^*$ by assumption, and the offense receives decreasing returns for i_m constant. That is, $\lambda(m) > \lambda(m+1)$, $m = 1,2,\ldots,R-2$. Thus some $A \leq R - 1$ is feasible and it is obvious that the minimum feasible A dictates the least cost firing doctrine. Then

$$I = \sum_{m=1}^{R-1} i_m$$

and

$$I = 2A + R - 1 - A = A + R - 1.$$

The payoff curve facing the offense is shown in Figure 9. Thus the optimal attack is $r^0 = R$ with possible alternate optima at $r^0 = 1$ and $r^0 = r_4$. In any case, r^0 yields λ^*.

D. LARGE TARGETS

Now suppose $(1-p)^3V \leq \lambda^* < (1-p)^2V$. Reasoning as above, note that this city is too valuable in relation to λ^* for the defense to fire only two interceptors at the first R.V. Notice that if $i_1 = 2$ then $r = 1$ yields $\lambda = (1-p)^2V > \lambda^*$. Thus the defense must use the firing doctrine $FD = (i_1,\ldots i_{R-1})$; $i_m = 3$, $m = 1,2,\ldots,B$; $i_m = 2$, $m = B+1,\ldots,A$; $i_m = 1$, $m = A+1,\ldots,R-1$. Notice that $B = R - 1$ is feasible since against this doctrine; $r = 1$ yields $\lambda = (1-p)^3V < \lambda^*$ and the offense receives decreasing returns. Thus there is some $B \leq R - 1$ that is feasible. It is appealing to think that the minimum

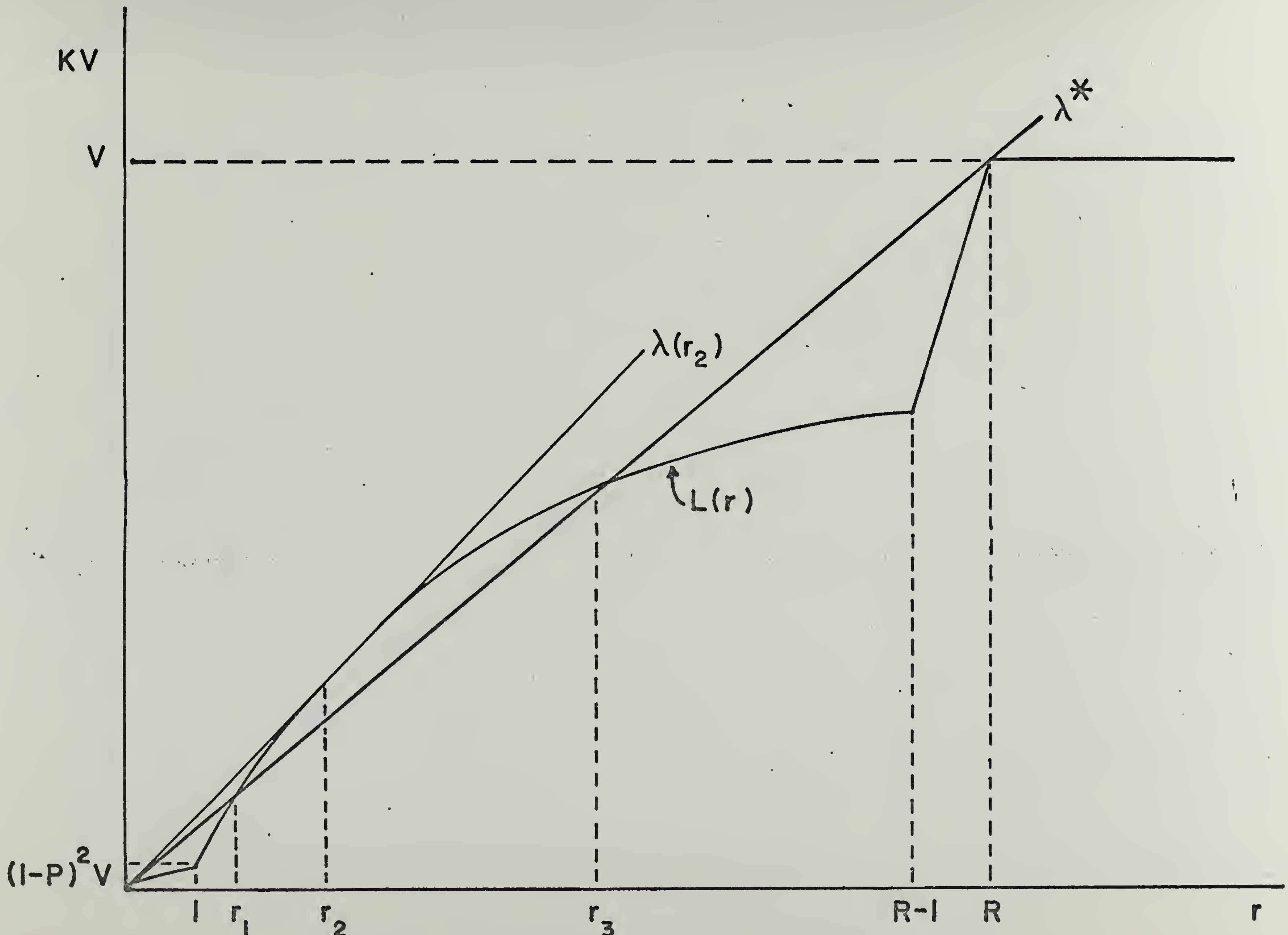

Figure 8. Non-Feasible Payoff Curve for Medium Target.

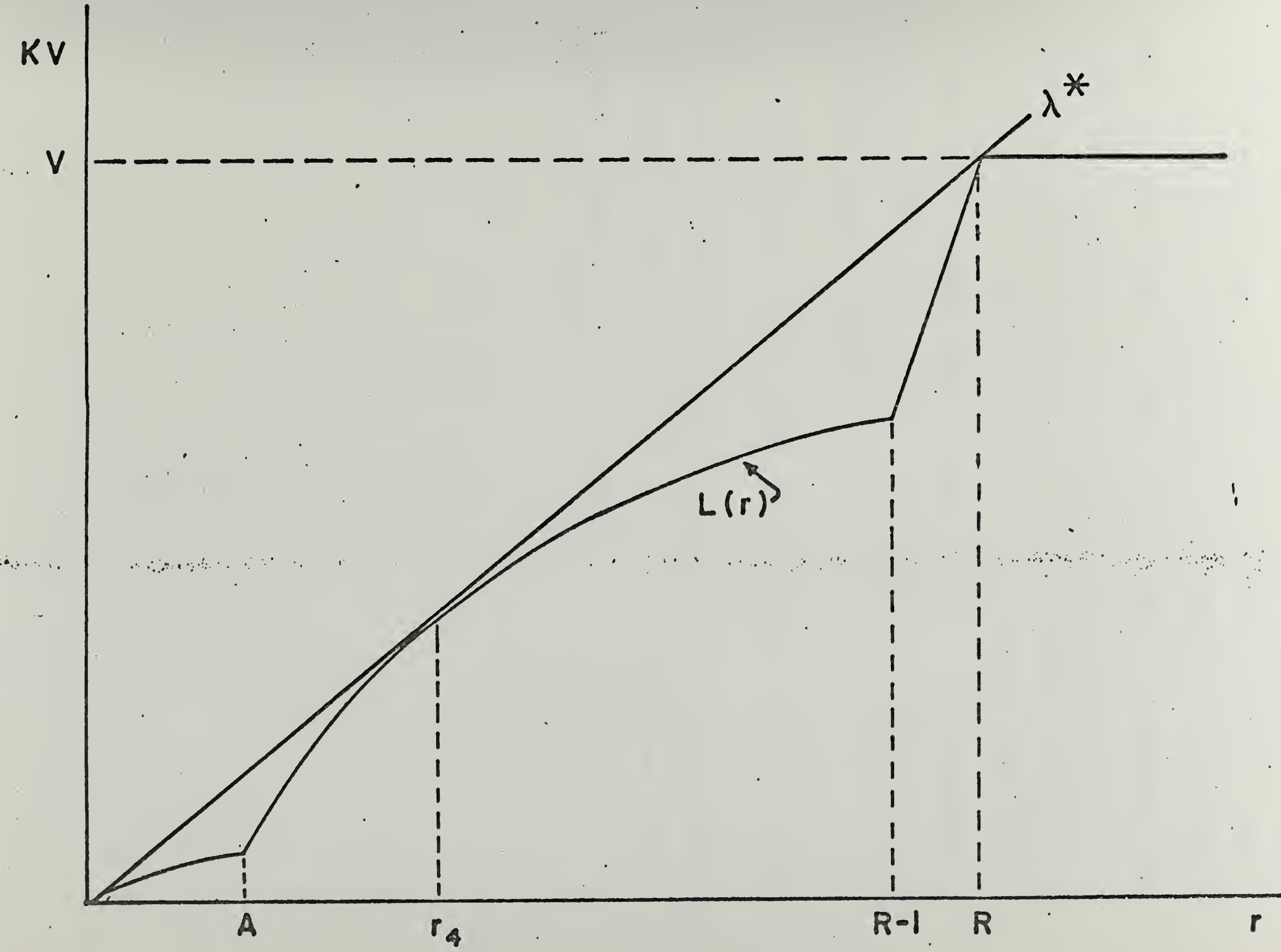

Figure 9. Optimal Payoff Curve for Medium Target.

B that is feasible is optimal. It can be shown that this is in fact the case. Also, for a fixed B, there is some $A \le R - 1$ that is feasible and the minimum A that satisfies the constraint must be least cost. Thus the defense fires three interceptors at the first B R.V.'s, where B is as small as possible; then fires two interceptors at the next A - B R.V.'s, where A is as small as possible; and then fires one interceptor up to the $(R - 1)^{st}$ R.V. at which point the interceptor supply is exhausted. Values of A and B can be easily computed. Then

$$I = \sum_{m=1}^{R-1} i_m = 3B + 2(A-B) + R - 1 - A - B = B + A + R - 1.$$

Figure 10 illustrates this last case. It is clear that again the optimal offense strategy is $r^0 = R$, with three alternate optima. And of course r^0 yields λ^*.

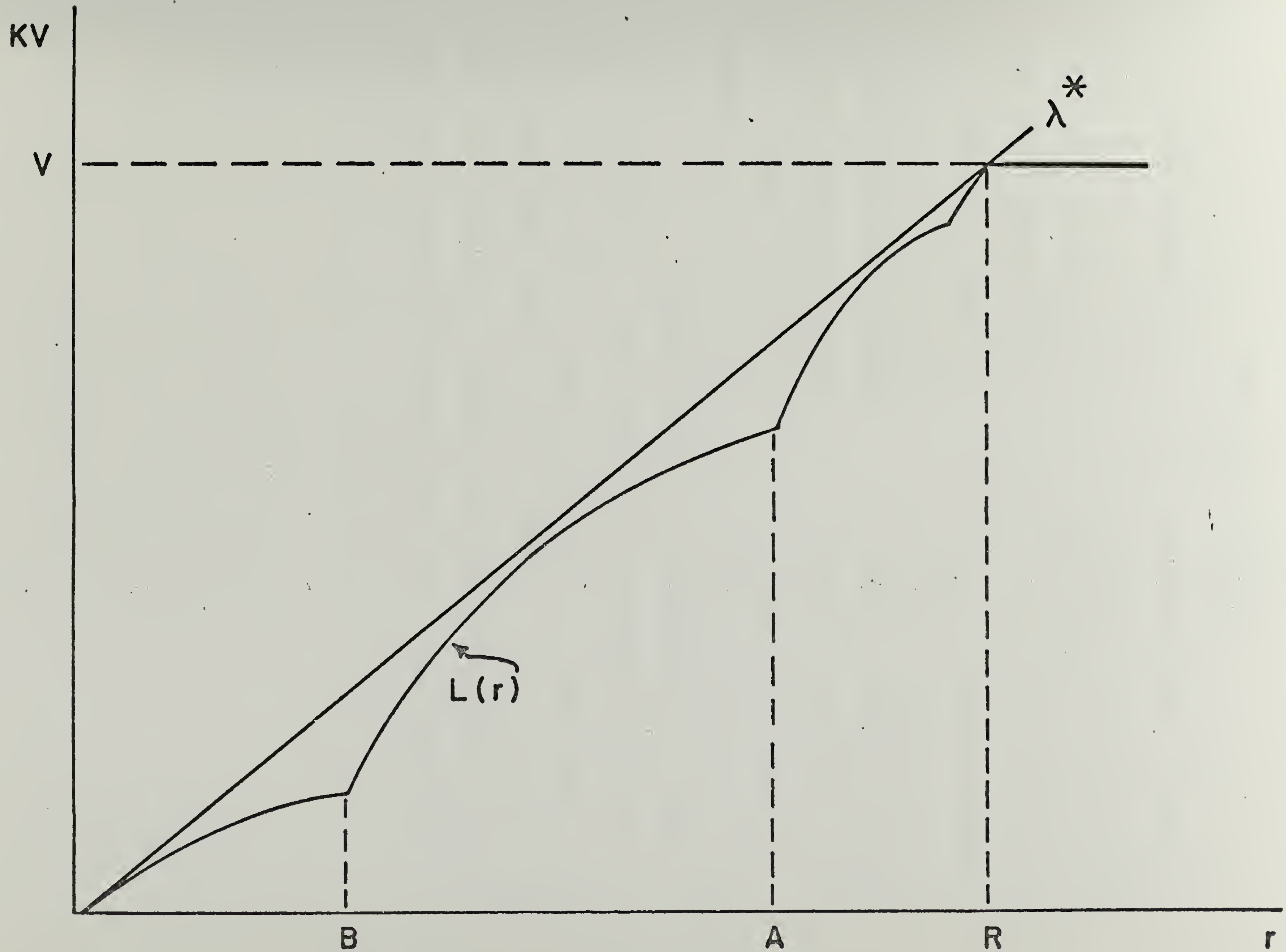

Figure 10. Optimal Payoff Curve for Large Target.

V. NUMERICAL PROCEDURES

The determination of the firing doctrine for small targets is trivial, and the firing doctrine for medium targets is a special case of that for large targets. The large target algorithm is shown in Figure 11. The algorithm is simple and quite fast. The simplicity of the determination depends upon the fact that the minimum B that allows a feasible solution minimizes I. This can be shown by letting B^0 be the smallest value of B such that feasibility can be maintained. Given B^0, let A^0 be the smallest feasible value of A. Denote the firing doctrine determined by B and A by F(B,A). Suppose $B^* = B^0 + 1$ allowed us to reduce I and be feasible. Then $F(B^*,A^*)$ is feasible, where $A^* = A^0 - 2$. It can be shown that if $F(B^0,A^0)$ and $F(B^*,A^*)$ are feasible, then $F(B^*-1,A^*+1) = F(B^0,A^0-1)$ is feasible. But $F(B^0,A^0-1)$ can't be feasible since A^0 was defined to be the minimum A which maintains feasibility. Thus $F(B^*,A^*)$ is not feasible. The proof is similar for $B^* = B^0 + c$.

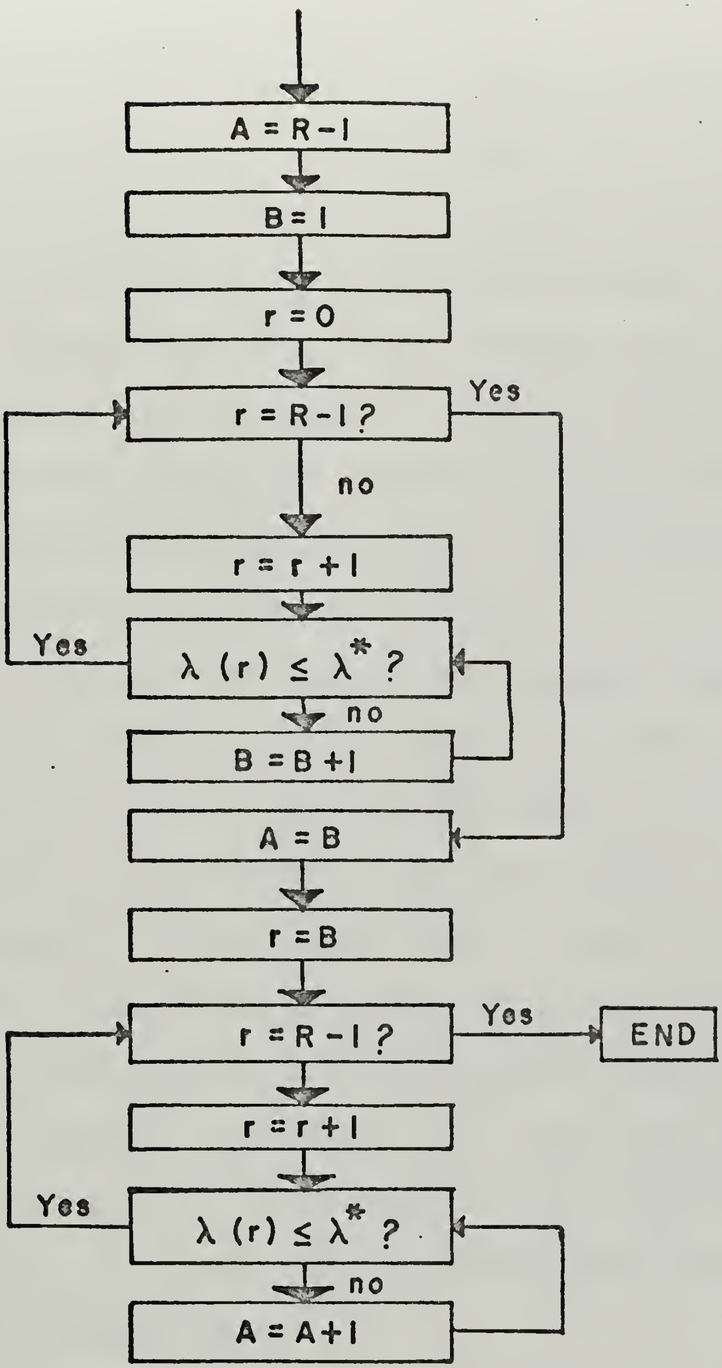

Figure 11. Large City Flow Chart.

VI. AN EXAMPLE

For any level of allowable offensive payoff, targets of value close to λ^* will be allocated only a few interceptors. Considering costs of land, radar, etc., it is not an efficient policy to set up a defensive complex for the purpose of defending against a small number of re-entry vehicles. One way to "solve" this problem is to arbitrarily agree to defend a certain number of targets, or equivalently to defend targets above a certain value. The defense is of course no longer exactly proportional.

The defense of 100 targets of various values is considered in the example. The target list used approximates the target structure of the United States, with all cities over 150,000 being defended and large cities broken into several targets. No consideration is taken of overlapping interceptor coverage or of target vulnerability, population values are approximate, and population is the only measure of value. Parametric studies of λ^* and p were conducted to determine changes in the firing doctrine and allocation of interceptors to targets, and to investigate total offense and defense requirements.

Table I shows the firing doctrine for 10 selected combinations of target value λ^*, and single shot kill probability p. Table II shows the total defensive interceptor requirement for various values of λ^* and p. Table II also shows, for various values of λ^*, the total number of re-entry vehicles

TABLE I

ALLOCATION AND FIRING DOCTRINE FOR SELECTED TARGETS

TARGET	VALUE·10^{-3}	p	$\lambda^*\cdot 10^{-3}$	Fire 3 at first	Fire 2 at next	Fire 1 at next	I	R
A	2647	.80	22	71	40	9	302	121
B	2647	.90	22	3	93	24	219	121
C	1424	.80	48	1	20	8	51	30
D	800	.90	22	0	16	20	52	37
E	647	.80	22	1	20	8	51	30
F	647	.90	22	0	11	18	40	30
G	450	.92	24	0	2	16	20	19
H	422	.90	24	0	3	14	20	18
I	422	.92	24	0	2	15	19	18
J	410	.90	24	0	3	14	20	18
K	380	.80	26	0	6	8	20	15

TABLE II

DEFENSIVE AND OFFENSIVE REQUIREMENTS FOR 100 TARGET PROBLEM

λ^*	ΣR	p= .8	.9	.92	.94	.96
22	2211	3778	2895	2735	2554	2364
26	1879	3040	2355	2229	2085	1943
30	1630	2537	1969	1862	1754	1644
40	1231	1727	1373	1310	1244	1178
50	997	1296	1048	1001	957	918
		TOTAL DEFENSIVE INTERCEPTOR REQUIREMENT				

ΣR required by the offense to attack all 100 targets with exhausting attacks of size $r = R$.

Since R is now required to be an integer, $\lambda(R)$ is typically less than λ^*, and it is no longer the case that the optimal offensive strategy is to use $r = R$. For example at target K (see Table I), R = 15 but the optimal attack has been computed to be $r^0 = 10$. The payoff for the exhausting attack is $\lambda(R) = 25{,}330$ and the payoff for the optimal attack is $\lambda(r^0) = 25{,}960$. The attack $r = 10$ is equivalent to $r = r_4$ in Figure 6. Notice however that the payoff with $r = r^0$ is less than λ^* as required and is only slightly greater than $\lambda(R)$. Furthermore, if the defense desired that only exhausting attacks be optimal (so that the offense must attack fewer targets), only small increases in A and B will be required to force the offense to set $r = R$ at all targets.

VII. GAME THEORY

The assumption that the offense knows the defensive firing doctrine has been useful to easily derive an interceptor allocation and firing doctrine. The offense however will generally not have this knowledge. The usefulness of the model then depends upon how closely the minmax model approximates the game theoretic model. Everett [5] shows that for a fixed allocation of interceptors at a target, the minmax approach allows an offensive payoff that is typically no more than 10% greater than the mixed strategy payoff.

Note that those targets where $i_m = 1$ for all m the game theoretic solution has been attained: minmax = maxmin. At targets where $A > 0$ however, game theory solutions can be used to reduce the allocation of interceptors at the target. The game is a simple, but large, matrix game.

VIII. CONCLUSIONS

The proportional minmax allocation model minimizes the number of interceptors required to force the offensive payoff (in terms of expected value per re-entry vehicle) to be less than or equal to a fixed payoff λ^*. Under the assumption that many defended targets will not be attacked, such an allocation will minimize the total value destroyed by the offense. By appropriate choice of λ^* the model determines the defensive allocation which minimizes total expected damage for a fixed number of interceptors.

The model has several attractive features. First, it allocates interceptors so that price is proportional to value. This type of defense is attractive since each city is equally defended. That is, the proportional allocation forces the offense to be indifferent as to which subset of defended targets he will attack. Furthermore, the minmax assumption allows the defense to easily generate interceptor allocations for various values of p and λ^*. Finally, the conceptual simplicity of the proportional defense model provides insight into the way costs vary in relation to levels of defense and defensive missile effectiveness.

LIST OF REFERENCES

1. Stanford Research Institute Technical Report No. 5205-20, A Methodology for National Deployment of Local Ballistic Missile Defense Batteries (U), by G. A. Branch, June 1966.

2. The RAND Corporation, RM-5943-PR, Some Implications of Leakage for Self-Defense of an ABM System (U), by R. D. Shaver, April 1969.

3. Stanford Research Institute Technical Note SED TN 194, The Uniform AVDAM Deployment Method, by C. T. Battle, September 1967.

4. Danskin, J. M., "On Suboptimization: An Example," Operations Research 15, p. 680-681, 1967.

5. The Lambda Corporation Paper 17, Defense Models XI, Exhaustion of Interceptor Supplies, by H. Everett, III, April 1968.

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13. ABSTRACT

This thesis presents a ballistic missile defense allocation model for the terminal defense of urban targets of varied value. The model allocates interceptors in proportion to the value of targets. Defensive missiles have a probability of interception, offensive re-entry vehicles are perfect, and the offense knows both the defensive allocation and firing doctrine. The area defended by a single interceptor farm is considered to be a point target and can be defended by no other interceptors. For any value of the offensive payoff in expected value per re-entry vehicle, the model determines the least cost minmax allocation and firing doctrine.

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